Journal of Nepal Mathematical Society (JNMS)
ISSN: 2616-0153 (Print), 2616-0161 (Online)
Vol. 8, Issue 1, 2025 (June): 76-82
DOI: https://doi.org/10.3126/jnms.v8i1.80308
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Research Article
Received Date: April 5, 2025
Accepted Date: May 6, 2025
Published Date: June 25, 2025

On Some Difference Sequence Spaces of Bi-complex Numbers

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Abstract: *In this paper, we study difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ in the context of bicomplex numbers. We show that these spaces form a linear vector space. Further, we explore their algebraic, topological, and geometric properties. Several illustrative examples are also provided to support the results.*
Keywords: Bi-complex numbers, Difference sequence space, Solidness, Completeness, Convexity.

1 Introduction

Bi-complex numbers are the generalization of complex numbers by introducing two distinct imaginary units. The idea was first introduced by the Italian mathematician Corrado Segre in 1892 [15]. The most detailed exploration of bi-complex numbers was later conducted by G. B. Price [12]. Additionally, Alpay et al. [1] developed a broad framework of functional analysis based on bi-complex scalars.

Several other researchers, including Bera and Tripathy [2, 3], Degirmen and Sagir [4, 5], Parajuli et al. [11], Rochon and Shapiro [13], Sager and Sagir [14], and Wagh [16] have studied the algebraic, geometric, and topological aspects of bi-complex sequence spaces.

In this article, we denote the sets of real, complex, and bi-complex numbers by the symbols $\mathbb{C}_0$, $\mathbb{C}_1$, and $\mathbb{C}_2$, respectively.
A bi-complex number [15] z is defined as:

$$z = (a + ib) + j(c + id) = z_1 + jz_2,$$

where $z_1 = a + ib$, $z_2 = c + id$, and $a, b, c, d \in \mathbb{C}_0$, $z_1, z_2 \in \mathbb{C}_1$.

The imaginary units satisfy the relations:

$$i^2 = j^2 = -1, \quad ij = ji = k, \quad \text{where } k \text{ is a hyperbolic unit such that } k^2 = 1.$$

The set of bi-complex numbers is denoted by:

$$\mathbb{C}_2 = \{z_1 + jz_2 : z_1, z_2 \in \mathbb{C}_1\}.$$

Apart from 0 and 1, the set $\mathbb{C}_2$ contains exactly two non-trivial idempotent elements, denoted by e_1 and e_2, which are defined as follows:

$$e_1 = \frac{1 + ij}{2}, \quad e_2 = \frac{1 - ij}{2}.$$

These elements satisfy the following properties:

$$e_1 + e_2 = 1, \quad e_1 - e_2 = ij, \quad e_1.e_2 = e_2.e_1 = 0, \quad e_1^2 = e_1, \quad e_2^2 = e_2.$$

Every bi-complex number $z = z_1 + jz_2$ (where $z_1, z_2 \in \mathbb{C}_1$) can be uniquely expressed as:

$$z = \mu_1 e_1 + \mu_2 e_2,$$

where

$$\mu_1 = z_1 - iz_2, \quad \mu_2 = z_1 + iz_2,$$

represent the idempotent components of z. The set $\{e_1, e_2\}$ serves as an orthogonal basis of $\mathbb{C}_2$.

The Euclidean norm on $\mathbb{C}_2$ is defined by

$$\|z\|_{\mathbb{C}_2} = \sqrt{a^2 + b^2 + c^2 + d^2} = \sqrt{|z_1|^2 + |z_2|^2} = \sqrt{\frac{|\mu_1|^2 + |\mu_2|^2}{2}}.$$

A sequence space refers to a vector space composed of elements that are infinite sequences. Several researchers, including Et [6], Ghimire and Pahari [7], Kamthan and Gupta [9], and Pahari [10], have studied various aspects of the theory of scalar and vector-valued sequence spaces using Banach sequences. Sequences may be real, complex, or bi-complex numbers.

In this article, we use the symbols $\omega(\mathbb{C}_2)$, $\ell_\infty(\mathbb{C}_2)$, $c(\mathbb{C}_2)$, and $c_0(\mathbb{C}_2)$ to represent the spaces of all, bounded, convergent, and null sequences of bi-complex numbers, respectively. These spaces are equipped with the norm defined by

$$\|z\|_\infty = \sup_k \|z_k\| \quad \text{for all } k \in \mathbb{N},$$

where $z = (z_k)$ is a sequence of bi-complex numbers. In order to explore the generalized limit of divergent sequences, Kizmaz [8] developed the notion of difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$, and $c_0(\Delta)$ for sequences of real or complex numbers as follows:

$$Z(\Delta) = \{z = (z_k) : \Delta z_k \in Z\} \quad \text{for } Z = \ell_\infty, c, \text{ or } c_0,$$

where $\Delta z = \Delta z_k = z_k - z_{k+1}$ for $k \in \mathbb{N}$.

In this work, we extend the above sequence spaces to the context of bi-complex numbers, defined by

$$\ell_\infty(\mathbb{C}_2)(\Delta) = \{z = (z_k) : \Delta z_k \in \ell_\infty(\mathbb{C}_2)\},$$

$$c(\mathbb{C}_2)(\Delta) = \{z = (z_k) : \Delta z_k \in c(\mathbb{C}_2)\}.$$

$$c_0(\mathbb{C}_2)(\Delta) = \{z = (z_k) : \Delta z_k \in c_0(\mathbb{C}_2)\}.$$

Obviously,

$$c_0(\mathbb{C}_2)(\Delta) \subset c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta).$$

The proof is straightforward. If $z_k \in c_0(\mathbb{C}_2)(\Delta)$, then $\|\Delta z_k\|_{\mathbb{C}_2} \to 0$ as $k \to \infty$. So, the sequence z_k is convergent and bounded. Hence,

$$z_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta).$$

Therefore,

$$c_0(\mathbb{C}_2)(\Delta) \subset c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta).$$

For example, let $z_k = \dfrac{1}{k}\mathbf{j}$, where $k \in \mathbb{N}$ and j is an imaginary unit. Then,

$$\Delta z_k = z_k - z_{k+1} = \left(\frac{1}{k} - \frac{1}{k+1}\right)\mathbf{j} = \frac{1}{k(k+1)}\mathbf{j}.$$

Clearly, $\Delta z_k \to 0$ as $k \to \infty$.

Thus, Δz_k is null, convergent, and bounded. So $z_k \in c_0(\mathbb{C}_2)(\Delta)$, and hence also $z_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.

2 Definitions and Preliminaries

Definition 2.1: A sequence in $\mathbb{C}_2$ is a function

$$z : \mathbb{N} \to \mathbb{C}_2,$$

represented as $z = (z_k)$, where each $z_k \in \mathbb{C}_2$.
A sequence (z_k) converges to $z \in \mathbb{C}_2$ if for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$\|z_k - z\|_{\mathbb{C}_2} < \varepsilon, \quad \forall k \geq N.$$

It can be written as

$$\lim_{k\to\infty} z_k = z.$$

A sequence (z_k) is called a Cauchy sequence if for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$\|z_m - z_n\|_{\mathbb{C}_2} < \varepsilon, \quad \forall m, n \geq N.$$

Definition 2.2: A sequence space X is solid (or normal) if

$$(z_k) \in X \implies (\alpha_k z_k) \in X$$

for any scalar sequence (α_k) with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$.
The space X is symmetric if for any permutation $\pi : \mathbb{N} \to \mathbb{N}$,

$$(z_k) \in X \implies (z_{\pi(k)}) \in X.$$

Also, the space X is monotone if it consists of canonical pre-images of all its step spaces.

Definition 2.3: A subset Y of a linear space X is said to be convex if for all $x, y \in Y$ and for all scalar $\lambda \in [0, 1]$,

$$(1 - \lambda)x + \lambda y \in Y.$$

3 Main Results

In this section, we present some theorems and examples exploring some algebraic, topological and geometric properties of the difference sequences of bi-complex numbers.

Theorem 3.1. *The sequence spaces $Z[\Delta, \|\cdot\|_{\mathbb{C}_2}]$, where $Z = \ell_\infty(\mathbb{C}_2)$, $c(\mathbb{C}_2)$, $c_0(\mathbb{C}_2)$, are normed linear spaces.*

Proof. We show that $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$ is a normed linear space. For this, let $y = (y_k)$ and $z = (z_k)$ be two arbitrary sequences in the space $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$. Then, we have

$$\sup_k \|\Delta y_k\|_{\mathbb{C}_2} < \infty \quad \text{and} \quad \sup_k \|\Delta z_k\|_{\mathbb{C}_2} < \infty.$$

Let $\alpha, \beta \in \mathbb{C}_0$. Then

$$\begin{aligned}
\sup_k \|\Delta(\alpha y_k + \beta z_k)\|_{\mathbb{C}_2} &\leq \sup_k \|\Delta(\alpha y_k) + \Delta(\beta z_k)\|_{\mathbb{C}_2} \\
&\leq \sup_k \left(\|\Delta(\alpha y_k)\|_{\mathbb{C}_2} + \|\Delta(\beta z_k)\|_{\mathbb{C}_2}\right) \\
&= |\alpha| \sup_k \|\Delta y_k\|_{\mathbb{C}_2} + |\beta| \sup_k \|\Delta z_k\|_{\mathbb{C}_2} \\
&< \infty.
\end{aligned}$$

Therefore, $(\alpha y_k + \beta z_k) \in \ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$.
Hence, $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$ is a linear space. We can establish the linearity of other spaces by a similar procedure.

Next, we prove that $\|\cdot\|_\Delta$ is a norm on $Z[\Delta, \|\cdot\|_{\mathbb{C}_2}]$, defined by

$$\|z\|_\Delta = \|z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta z_k\|_{\mathbb{C}_2},$$

where $Z = \ell_\infty(\mathbb{C}_2),\ c(\mathbb{C}_2),\ c_0(\mathbb{C}_2)$ and $z = (z_k)$.

(i) Let $z = \theta$, where $\theta = (0, 0, \dots)$ is the zero element of Z. Then,

$$\|\theta\|_\Delta = \|0\|_{\mathbb{C}_2} + \sup_k \|\Delta 0\|_{\mathbb{C}_2} = 0 + 0 = 0.$$

Conversely, suppose

$$\|z\|_\Delta = \|z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta z_k\|_{\mathbb{C}_2} = 0.$$

Then $\|z_1\|_{\mathbb{C}_2} = 0 \Rightarrow z_1 = 0$, and

$$\|\Delta z_k\|_{\mathbb{C}_2} = 0 \quad \text{for all } k \in \mathbb{N}.$$

For $k = 1$, we have

$$\|\Delta z_1\| = \|z_1 - z_2\| = \|0 - z_2\| = \|z_2\| = 0 \Rightarrow z_2 = 0.$$

Proceeding in this way, we find $z_k = 0$ for all $k \in \mathbb{N}$. Therefore, $\|z\|_\Delta = 0 \iff z = 0$.

(ii) Let $y = (y_k)$ and $z = (z_k)$. Then,

$$\begin{aligned}
\|y + z\|_\Delta &= \|y_1 + z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta(y_k + z_k)\|_{\mathbb{C}_2} \\
&\le (\|y_1\|_{\mathbb{C}_2} + \|z_1\|_{\mathbb{C}_2}) + \left(\sup_k \|\Delta y_k\|_{\mathbb{C}_2} + \sup_k \|\Delta z_k\|_{\mathbb{C}_2}\right) \\
&= \left(\|y_1\|_{\mathbb{C}_2} + \sup_k \|\Delta y_k\|_{\mathbb{C}_2}\right) + \left(\|z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta z_k\|_{\mathbb{C}_2}\right) \\
&= \|y\|_\Delta + \|z\|_\Delta.
\end{aligned}$$

Thus, $\|y + z\|_\Delta \le \|y\|_\Delta + \|z\|_\Delta$.

(iii) Let $\alpha \in \mathbb{C}_0$ and $z = (z_k) \in \mathbb{C}_2$. Then,

$$\begin{aligned}
\|\alpha z\|_\Delta &= \|\alpha z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta(\alpha z_k)\|_{\mathbb{C}_2} \\
&= |\alpha| \|z_1\|_{\mathbb{C}_2} + |\alpha| \sup_k \|\Delta z_k\|_{\mathbb{C}_2} \\
&= |\alpha| \left(\|z_1\|_{\mathbb{C}_2} + \sup_k \|\Delta z_k\|_{\mathbb{C}_2}\right) \\
&= |\alpha| \|z\|_\Delta.
\end{aligned}$$

Thus, $\|\alpha z\|_\Delta = |\alpha| \|z\|_\Delta$.

Hence, $\|\cdot\|_\Delta$ is a norm on $Z = \ell_\infty(\mathbb{C}_2),\ c(\mathbb{C}_2),\ c_0(\mathbb{C}_2)$.
Therefore, $\ell_\infty(\mathbb{C}_2)[\Delta],\ c(\mathbb{C}_2)[\Delta],\ c_0(\mathbb{C}_2)[\Delta]$ are normed linear spaces. □

Theorem 3.2. *The class of sequence spaces* $Z[\Delta, \|\cdot\|_{\mathbb{C}_2}]$*, where* $Z = \ell_\infty(\mathbb{C}_2),\ c(\mathbb{C}_2),\ c_0(\mathbb{C}_2)$ *are Banach spaces.*

Proof. Let (z^n) be any Cauchy sequence in $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$, where

$$z^n = (z_k^n) = \{z_1^n, z_2^n, z_3^n, \dots\} \in \ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}], \quad n \in \mathbb{N}.$$

Then,

$$\|z^n - z^m\|_\Delta = \|z_1^n - z_1^m\| + \sup_k \|\Delta z_k^n - \Delta z_k^m\| \to 0 \quad \text{as } n, m \to \infty.$$

Thus, we obtain

$$\|z_k^n - z_k^m\| \to 0 \quad \text{as } n, m \to \infty \quad \text{for each } k \in \mathbb{N}.$$

Hence, the sequence $(z_k^n) = \{z_k^1, z_k^2, z_k^3, \ldots\}$ forms a Cauchy sequence in $\mathbb{C}_2$. Since $\mathbb{C}_2$ is complete [14], it converges to z_k say. Therefore,

$$z_k^n \to z_k \quad \text{as } n \to \infty.$$

Moreover, for every $\varepsilon > 0$ there exists a positive integer $N = N(\varepsilon)$ such that for all $n, m \geq N$ and for all $k \in \mathbb{N}$, we have

$$\|z_1^n - z_1^m\| < \varepsilon, \quad \text{and} \quad \|(z_{k+1}^n - z_{k+1}^m) - (z_k^n - z_k^m)\| < \varepsilon.$$

Also,

$$\lim_{m\to\infty} \|z_1^n - z_1^m\| = \|z_1^n - z_1\| \leq \varepsilon,$$

and for all $n \geq N$,

$$\lim_{m\to\infty} \|(z_{k+1}^n - z_{k+1}^m) - (z_k^n - z_k^m)\| = \|(z_{k+1}^n - z_{k+1}) - (z_k^n - z_k)\| \leq \varepsilon.$$

Since ε is independent of k, we have

$$\sup_k \|(z_{k+1}^n - z_{k+1}) - (z_k^n - z_k)\| \leq \varepsilon.$$

Consequently, for $n \geq N$,

$$\|z^n - z\|_\Delta \leq 2\varepsilon.$$

Hence,

$$z^n \to z \quad \text{as} \quad n \to \infty,$$

where $z = (z_k)$.
Now we show that $z \in \ell_\infty(\mathbb{C}_2)[\Delta]$. We have

$$\|z_k - z_{k+1}\| = \|z_k - z_k^N + z_k^N - z_{k+1}^N + z_{k+1}^N - z_{k+1}\| \leq \|z_k^N - z_{k+1}^N\| + \|z^N - z\|_\Delta = O(1),$$

which shows that$z = (z_k) \in \ell_\infty(\mathbb{C}_2)[\Delta]$.
Since every Cauchy sequence in $\ell_\infty(\mathbb{C}_2)[\Delta]$ converges in $\ell_\infty(\mathbb{C}_2)[\Delta]$, it is complete. Also, being a normed linear space, it is a Banach space.

Likewise, using the same procedure as above, we can show that the spaces $c(\mathbb{C}_2)[\Delta]$ and $c_0(\mathbb{C}_2)[\Delta]$ are Banach spaces.

□

Theorem 3.3. *The sequence space* $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$ *is convex.*

Proof. Let $y = (y_k)$ and $z = (z_k) \in \ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$ and $\lambda \in \mathbb{C}_0$ satisfying $0 \leq \lambda \leq 1$.
Then,

$$\sup_k \|\Delta y_k\|_{\mathbb{C}_2} < \infty \quad \text{and} \quad \sup_k \|\Delta z_k\|_{\mathbb{C}_2} < \infty.$$

Now,

$$\begin{aligned}\sup_k \|\Delta\{\lambda y_k + (1-\lambda) z_k\}\|_{\mathbb{C}_2} &\leq \sup_k \|\Delta\{\lambda y_k\}\|_{\mathbb{C}_2} + \sup_k \|\Delta\{(1-\lambda) z_k\}\|_{\mathbb{C}_2} \\ &= \lambda \sup_k \|\Delta y_k\|_{\mathbb{C}_2} + (1-\lambda) \sup_k \|\Delta z_k\|_{\mathbb{C}_2} \\ &< \infty + \infty = \infty.\end{aligned}$$

Therefore, $\{\lambda y_k + (1-\lambda) z_k\} \in \ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$.
Hence, $\ell_\infty(\mathbb{C}_2)[\Delta, \|\cdot\|_{\mathbb{C}_2}]$ is convex.

A similar procedure can be used to establish the result for the sequence spaces $c(\mathbb{C}_2)[\Delta]$ and $c_0(\mathbb{C}_2)[\Delta]$.

□

Example 1. *Let $z_k = (k+1)j$ where k is a positive integer and j is an imaginary unit. The sequence (z_k) does not converge in $\mathbb{C}_2$. So, $z_k \notin c_0(\mathbb{C}_2) \subset c(\mathbb{C}_2) \subset \ell_\infty(\mathbb{C}_2)$. But*

$$\Delta z_k = z_k - z_{k+1} = (k+1)j - (k+2)j = -j$$

which is bounded as well as convergent. Hence, $z_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.

Example 2. *Let $z_k = (e_1 + e_2)kj$ where k is a positive integer.*
Then, $\Delta z_k = -(e_1 + e_2)j$. So, $z_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.
Let us define the sequence (α_k) of scalars:

$$\alpha_k = \begin{cases} 1 & \text{when } k \text{ is odd} \\ 0 & \text{when } k \text{ is even} \end{cases}$$

Then, $|\alpha_k| \leq 1$ for all positive integers $k \in \mathbb{N}$.
Now,

$$\alpha_k z_k = \begin{cases} (e_1 + e_2)kj & \text{when } k \text{ is odd} \\ 0 & \text{when } k \text{ is even} \end{cases}$$

Then,

$$\Delta \alpha_k z_k = \begin{cases} (e_1 + e_2)kj & \text{when } k \text{ is odd} \\ -(e_1 + e_2)kj & \text{when } k \text{ is even} \end{cases}$$

Hence, $\alpha_k z_k \notin c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.
Therefore, the spaces $c(\mathbb{C}_2)(\Delta)$ and $\ell_\infty(\mathbb{C}_2)(\Delta)$ are not solid.

Example 3. *Let $z_k = kj$ for all positive integers $k \in \mathbb{N}$. Then, $z_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.*
Consider the permutation $z_{\pi(k)}$ of the elements of z_k defined by

$$z_{\pi(k)} = \{z_1, z_2, z_4, z_3, z_9, z_5, z_{16}, z_6, z_{25}, \ldots\}.$$

Then, $z_{\pi(k)} \notin c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.
Hence, the spaces $c(\mathbb{C}_2)(\Delta)$ and $\ell_\infty(\mathbb{C}_2)(\Delta)$ are not symmetric.

Example 4. *Let $y_k = j$ for all positive integers $k \in \mathbb{N}$. Then $y_k \in c_0(\mathbb{C}_2)(\Delta)$.*
Consider the sequence (z_k) in its preimage space defined by

$$z_k = \begin{cases} j & \text{when } k \text{ is odd,} \\ 0 & \text{when } k \text{ is even.} \end{cases}$$

Then, $(z_k) \notin c_0(\mathbb{C}_2)(\Delta)$. Hence the space $c_0(\mathbb{C}_2)(\Delta)$ is not monotone.

Example 5. *Let $y_k = kj$ for all $k \in \mathbb{N}$. Then $y_k \in c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.*
Consider the sequence in its preimage space defined by

$$z_k = \begin{cases} j & \text{when } k \text{ is odd,} \\ 0 & \text{when } k \text{ is even.} \end{cases}$$

Now,

$$\Delta z_k = \begin{cases} j & \text{when } k \text{ is odd,} \\ -j & \text{when } k \text{ is even.} \end{cases}$$

Therefore, $z_k \notin c(\mathbb{C}_2)(\Delta) \subset \ell_\infty(\mathbb{C}_2)(\Delta)$.
Hence, $c(\mathbb{C}_2)(\Delta)$ and $\ell_\infty(\mathbb{C}_2)(\Delta)$ are not monotone.

4 Conclusion

The difference sequence spaces of bi-complex numbers exhibit various algebraic, topological, and geometric properties. In this study, we have studied some of these properties. There is also the possibility of extending these findings to generalized difference double sequences of bi-complex numbers in future research.

Acknowledgments

The authors would like to acknowledge and offer special thanks to the anonymous referee(s) for their suggestions and comments to improve the paper.

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